Testing temporal Bell inequalities through repeated measurements in rf-SQUIDs

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Abstract

Temporal Bell-like inequalities are derived taking into account the influence of the measurement apparatus on the observed magnetic flux in a rf-SQUID. Quantum measurement theory is shown to predict violations of these inequalities only when the flux states corresponding to opposite current senses are not distinguishable. Thus rf-SQUIDs cannot help to discriminate realism and quantum mechanics at the macroscopic level.

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When quantum mechanics is extended to the macroscopic world some contradictions with realism, *i.e.* the prejudice according to which objects exist regardless of their observation, are evident. A deeper understanding of this contrast has relevance both to better study quantum phenomena already occurring in the macroworld, such as macroscopic quantum transport of particles in superfluidity and superconductivity, and to understand the relationships among quantum mechanics, macroscopic realism and classical physics, this last being contained in the former but at the same time playing a crucial role for the existence of the measuring apparatus. It became evident that the relevant features under debate were testable with numerical predictions and actual experiments [1]. For instance, spatial Bell inequalities have been tested and the experimental results agreed with the violation of the inequalities predicted by quantum mechanics [2]. Although the interpretation of these results is still under debate [3], the attention has been shifted in recent years to test temporal Bell inequalities [4]. In this case the crucial difference is that a unique system undergoes to repeated measurements at different times, unlike the case of spatial Bell inequalities where two systems are subjected to unique and simultaneous measurements. Furthermore, the aim of temporal Bell inequalities, in the original spirit of Leggett and Garg [4], was to test quantum mechanics at the macroscopic level whenever a macroscopic observable of the system is monitored. This allows one to study the extension of quantum theory to the macroscopic world to solve its paradoxical contrast with the widely accepted realistic view [5,6]. Following this proposal, Tesche discussed in detail a concrete experimental scheme based upon use of superconducting quantum interferometer devices (SQUIDs) [7,8]. The feasibility of any experiment aimed at testing macroscopic realism through temporal Bell inequalities has been criticized due to the role played by the concept of non-invasive measurements [9,10]. In this letter we consider Bell inequalities for a measurement of magnetic flux on a rf-SQUID at certain set of times and the predictions of quantum theory including the effect of the previous measurements in the evolution of the system. We also consider the quantum limitations dictated by the uncertainty principle to the measurement of magnetic flux in the same set of measurements. The two investigations are finally merged together to estabilish if

theoretically *predicted* violations of temporal Bell inequalities can actually *be observed* when the effect of the measurement is taken into account.

The system we are considering is an rf-SQUID where the magnetic flux ϕ evolves in a bistable potential. The corresponding Hamiltonian for the magnetic flux ϕ (in the unit system in which $\hbar = 2m = 1$, m being the effective mass of the system) is:

$$H = -\frac{\partial^2}{\partial \phi^2} - \frac{\mu}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \tag{1}$$

where μ and λ ($\mu, \lambda > 0$) are parameters associated to the superconducting circuit. The potential corresponding to the last two terms in (1) has the shape of a double well with minima at $\pm\Phi_{min} = \pm(\mu/\lambda)^{1/2}$, separated by a distance $\Delta L \equiv 2\Phi_{min}$. The effective potential in (1) can be rewritten in terms of the minima and the energy barrier $|V(\Phi_{min})| = \mu^2/4\lambda$ as

$$V(\phi) = 2V(\Phi_{min}) \left[1 - \frac{1}{2}\left(\frac{\phi}{\Phi_{min}}\right)^2\right]\left(\frac{\phi}{\Phi_{min}}\right)^2. \tag{2}$$

Both the distance between the two minima ΔL and the energy barrier $|V(\Phi_{min})|$ depend upon the parameters μ and λ. The whole analysis is carried out in a dissipationless environment, in which quantum coherence can be observed. Following Leggett and Garg [4] we subdivide the values of magnetic flux in the two regions $\phi > 0$, $\phi < 0$, respectively corresponding to clockwise and counterclockwise senses for the superconducting currents. The probability for the observed magnetic flux Φ to correspond to one definite sense of circulation for the current, for instance $\Phi > 0$, is defined as

$$P\{\Phi(t) > 0\} = \frac{\int_0^{+\infty} d\phi |\psi(\phi, t)|^2}{\int_{-\infty}^{+\infty} d\phi |\psi(\phi, t)|^2} \tag{3}$$

where $\psi(\phi, t)$ is the time-dependent wavefunction of the superconducting current in the magnetic flux representation. It is possible to write also correlation probabilities for the results of two measurements performed at times t_i and t_j, with $t_{ij} = t_i - t_j$ called quiescent time (we consider the limit of impulsive measurements, having therefore a negligible duration, situation well approximated in practice by fast switching superconducting circuits), for instance

$$P_{+-}^{ij} \stackrel{\text{def}}{\equiv} P\{\Phi(t_i) > 0, \Phi(t_j) < 0\}. \tag{4}$$

In a realistic model, in which the sign of the flux is defined even when not measured, we can write temporal Bell-type inequalities such as

$$P_{+-}^{bc} \leq P_{++}^{ab} + P_{--}^{ac} \tag{5}$$

where different histories for the possible measurements have been considered: the magnetic flux not measured at t_a and measured respectively with positive and negative values at t_b and t_c, flux measured with both positive values at t_a and t_b and not measured at t_c, flux measured at t_a and t_c with both negative values and not measured at t_b (see Fig. 1). Eq. (5) can be rewritten in an alternative form, which shows the dependence on the quiescent times:

$$\Delta P(t_{ab}, t_{bc}) = P_{+-}^{bc} - P_{++}^{ab} - P_{--}^{ac} \leq 0. \tag{6}$$

We want to check whether quantum mechanics predicts violations of eq. (5),*i.e.* if exists at least a pair of quiescent times for which $\Delta P(t_{ab}, t_{bc}) > 0$.

The effect of the measurement process is introduced by means of a non-unitary filtering weight which selects a particular result of the measurement with a given accuracy. In this way the wavefunction at the end of an impulsive measurement $\psi(\phi, t^+)$ is given by the wavefunction immediately before the measurement $\psi(\phi, t^-)$ multiplied by a weight function $w_\Phi(\phi)$. The square modulus of the output wavefunction $\psi(\phi, t^+)$ is the probability of finding the system in the state given by $w_\Phi(\phi)$ itself. Following von Neumann [11] we write such a weight as

$$w_\Phi^{v.N.}(\phi) \propto \begin{cases} 1 & \text{if } |\phi - \Phi| < \Delta\Phi, \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where $2\Delta\Phi$, the width of the filter of the meter, is hereafter called instrumental error. Other choices for the filtering weight are possible. For instance a less discontinuous, and therefore more physical, weight function is written, as in [12]:

$$w_\Phi(\phi) \propto \exp\left\{-\frac{(\phi-\Phi)^2}{2\Delta\Phi^2}\right\} \tag{8}$$

where $\Delta\Phi^2$ assumes the meaning of a variance. Also, a filter complementary to (7), which would leave unchanged the state only if the magnetic flux is localized around Φ, is the analytical counterpart of the so called null-result measurement scheme proposed in [8]. In either choices a particular outcome is privileged with respect to the other possible ones and this is reflected in the dynamical evolution of the magnetic flux. Moreover, the unitary evolution is broken during the measurement, as one expects for a selective measurement in which one get rid of all the possible alternatives incompatible with the measurement result. The actual value of the proportionality constants in eqs. (7) and (8) does not matter, because the only relevant quantities in the subsequent calculations are normalized probabilities. For instance the quantity

$$P(\Phi) = \frac{\parallel \psi_\Phi(t^+) \parallel^2}{\int \parallel \psi_{\Phi'}(t^+) \parallel^2 d\Phi'} = \frac{\parallel \psi_\Phi(t^+) \parallel^2}{\iint e^{-\frac{(\phi-\Phi')^2}{\Delta\Phi^2}} |\psi(\phi,t^-)|^2 d\phi\, d\Phi'} = \frac{1}{\sqrt{\pi}\Delta\Phi} \parallel \psi_\Phi(t^+) \parallel^2 \tag{9}$$

represents the probability that the observed value of the magnetic flux is Φ, with an instrumental error $\Delta\Phi$, in the case of a Gaussian weight function such as (8). It is also clear that, to distinguish the two signs of the magnetic flux required to have a dichotomic variable useful for building Bell inequalities, one has to work with instrumental errors $\Delta\Phi$ less than the distance between the two wells ΔL. We will consider in the following a system with fixed parameters μ and λ, and therefore constant ΔL, and variable instrumental error $\Delta\Phi$. This is equivalent to consider the opposite situation of a constant instrumental error and variables parameters of the rf-SQUID, since the relative magnitude between $\Delta\Phi$ and ΔL rules the distinguishability issue in a single measurement.

If more measurements are performed the back-action of the previous ones has to be taken into account and the distinguishability of the two signs of the magnetic flux depends, besides the instrumental error, upon the time intervals between consecutive measurements. Suppose that the system is initially in a pure state described by the wavefunction $\psi(\phi,0)$. Let us assume that a series of N measurements at $t_n \equiv nT$ $(n = 0, 1, \ldots, N-1)$, has been

performed with fixed instrumental error $\Delta\Phi$ and known results $\{\Phi_n\}$. Finally we suppose to perform another measurement at $t_N \equiv NT$. According to the (9), the probability for obtaining a result Φ_N in this last measurement is

$$P_{\{\Phi_n\}_{n\leq N-1}}(\Phi_N) = \frac{1}{\sqrt{\pi}\Delta\Phi} \parallel \psi_{\{\Phi_n\}_{n\leq N}}(t_N^+) \parallel^2, \tag{10}$$

i.e. it is proportional to the squared norm of the wavefunction after the N^{th} measurement. The analytical expression of this last is [13]

$$\psi_{\{\Phi_n\}_{n\leq N}}(\phi, t_N^+) = \sum_{l,m,n_1,\ldots,n_N=1}^{\infty} W_{mn_1}^{\Phi_N} W_{n_1 n_2}^{\Phi_{N-1}} \cdots W_{n_N l}^{\Phi_{min}} \exp\left\{-\frac{i\Delta T}{\hbar}\sum_{i=1}^{N} E_{n_i}\right\} c_l\, u_m(\phi) \tag{11}$$

where the E_i, u_i are respectively the energy eigenvalues and eigenstates of the system, the $W_{ij}^{\Phi}(\Delta\Phi)$'s are the matrix elements of $w_\Phi(\Phi)$ between energy eigenstates (expressed through (7) or (8) in terms of the instrumental error $\Delta\Phi$) on the latter and the c_l's are the projections on them of the initial state $\psi(\phi, 0)$. All the relevant quantities depend upon $\Delta\Phi$ through $W_{ij}^{\Phi}(\Delta\Phi)$ in eq. (11). If the effect of the measurement is taken into account in this way an effective magnetic flux uncertainty, with respect to the result $\tilde{\Phi}$, arises [14]

$$\Delta\Phi_{\mathit{eff}}(\{\Phi_n\}_{n\leq N-1}, N)^2 = 2\int_{-\infty}^{+\infty} (\Phi_N - \tilde{\Phi})^2 P_{\{\Phi_n\}_{n\leq N-1}}(\Phi_N)\, d\Phi_N. \tag{12}$$

The effective magnetic flux uncertainty takes into account, besides the instrumental error $\Delta\Phi$, the back-action effect of the previous measurements. For stroboscopic measurements with constant result, the effective uncertainty $\Delta\Phi_{\mathit{eff}}$ tends to reach an asymptotic value $\Delta\Phi_{\mathit{eff}}^{\text{as}}$ which is greater than the instrumental error $\Delta\Phi$, due to the effect of the back-action of the meter on the measured system, unless the system is monitored in a regime unaffected by the quantum noise, *i.e.* when $\Delta\Phi \gg \sigma$ where σ is the width of the initial wavefunction $\psi(\phi, 0)$, or in a quantum nondemolition way [15,16]. We have already identified the quiescent times T for which repeated measurements of flux are quasi-quantum nondemolition ones [13] as the multiples of the tunneling period $T = 2\pi\hbar/(E_2 - E_1)$. This is the reason why we have chosen T as the quiescent time for the preparatory sequence referred to in Fig. 1. The correlation probabilities (4) have been evaluated by applying (10), and choosing the

parameters of the potential in (1) as $\mu = 9.6$ and $\lambda = 1.536$ (always in the unit system in which $\hbar = 1$), such that $\Phi_{min} = 2.5$ and thus $\Delta L = 5$. The choice of the initial state $\psi(\phi, 0)$ is unessential because, after the optimal preparatory measurement sequence, the state collapses around the measurement result, as discussed in [14]. Now we can calculate the *quantum* predictions for ΔP using (3-6). In Fig. 2 a comparison between the results obtained for the temporal Bell inequality and the already-known spatial Bell inequality [1] is shown to be very similar in the dependence upon the relevant parameters, the quiescent times for the temporal case and the polarimeter angles for the spatial case.

An analogous dependence upon the measurement time (expressed in units of the tunneling period T) is shown in Fig. 3 for the effective magnetic flux uncertainties associated to each of the three sequences of measurement. The optimality is linked to the multiples of T: thus the different combinations of measurements are correlated to different orientations of the optimal regions in the (t_{ab}, t_{bc}) plane. For instance, in the case of sequence III of Fig. 1 (lowest plot in Fig. 3), there lie along diagonal lines, corresponding to $t_{ab} + t_{bc}$ multiple of the optimal periodicity T.

The exclusion among the regions of violation to Bell inequalities and the regions of distinguishability of the magnetic flux is emphasized in Fig. 4 which is a synthesis of all our discussion. Contour plots for the Bell inequality violation region, and for the regions of distinguishability of left and right part of the barrier for the sequences of Fig. 1, are simultaneously shown in a t_{ab}-t_{bc} plot. The shaded areas indicates the pairs of quiescent times for which $\Delta P(t_{ab}, t_{bc})$ is greater than zero, *i.e.* Bell inequalities are violated. The quasi-triangular regions correspond to the set of couples of quiescent times for which the two wells are resolved even after the measurements, *i.e.* all the three effective uncertainties $\Delta\Phi^{bc}_{+-}$, $\Delta\Phi^{ab}_{++}$ and $\Delta\Phi^{ac}_{--}$ are less than ΔL. No intersection among the various contours plots exists, *i.e.* for the sequences of measurements for which quantum mechanics gives predictions in contrast with that of a realistic theory, one cannot simply speak about distinct states because the effective uncertainty does not allow one to distinguish them. This result has been tested with respect to a certain number of conditions. Different values of the

instrumental uncertainty $\Delta\Phi$ have been chosen. Values of $\Delta\Phi$ larger than the intra-well separation ΔL do not allow to distinguish the two senses of the superconducting currents: optimal zones of distinguishability are present only for $\Delta\Phi < \Delta L/2$. Furthermore, for $\Delta\Phi > \Delta L$, the violations itself disappear. The plot has been obtained for some values of the instrumental error in a range of the order of the intra-well distance; moreover, the state has been prepared with different sequences of initial measurements. Also, both the filtering functionals (7) and (8) have been used. In all the examined cases, including $\Delta\Phi \ll \Delta L$, the results are qualitatively similar to the example shown in Fig. 4, as we will describe in detail in a future paper.

Our result, although obtained for a particular Bell inequality, should hold in general. Violations of temporal Bell inequalities stem from a subtle interplay between the request for resolving the two wells, to assign in an unambiguous way the sense of the superconducting current of the rf-SQUID, and the stringent demand for not destroying the coherence of the state during consecutive measurements which is at the basis of the superposition principle. Indeed the linearity of the quantum formalism permit superpositions of macroscopically distinct states which originates the difference from the realistic behaviour. Any reasonable quantum theory of measurement must introduce nonunitarity in the time evolution of a repeatedly observed system, destroying the abovementioned contradiction, as well illustrated by Feynman in the case of the two-slit experiment. Therefore violations to Bell inequalities are not observed either when no measurement is performed ($\Delta\Phi = \infty$) or when the measurement is too strong ($\Delta\Phi \to 0$). An intermediate regime exists in which violation of Bell inequalities is possible. Unfortunately even in this intermediate regime the violations are not centered, as already remarked in [4], around time intervals between consecutive measurements equal to multiple of the tunneling period. On the other hand, as discussed in detail in [13], the measurements are quantum nondemolition only for a periodicity equal to the tunneling period regardless of the particular shape of the bistable potential. With demolitive measurements instead, the back-action of the previous measurements has to be taken into account (as we have done by introducing the effective uncertainty $\Delta\Phi_{eff} \geq \Delta\Phi$)

ruling out the distinguishability of the two superconducting current senses. The Heisenberg principle, at the heart of quantum theory and based on classical considerations too, seems to protect Nature from observing contradictions between it and realism at the macroscopic level. As a consequence, even if in principle violations of temporal Bell-like inequalities are observable, they seem condemned to remain unobserved. This also requires a revision of the experiments aimed at testing temporal Bell inequalities proposed [8] and in preparation.

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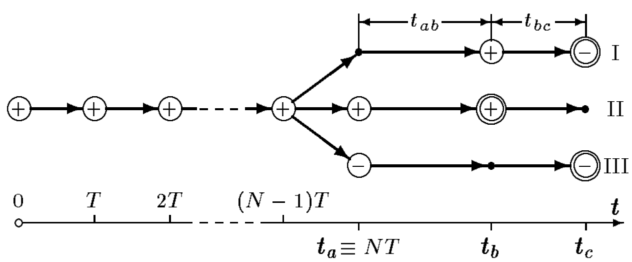
FIGURES

FIG. 1. Scheme of the simulated sequences of measurements for the calculation of the correlation probabilities in (6). After a preparatory sequence of $N = 16$ measurements with the optimal periodicity $T = 2\pi\hbar/(E_2 - E_1)$ and constant results $\Phi_n \equiv -\Phi_{min}$ (such that $\Delta\Phi_{eff}$ has reached its asymptotic value, as stated in [14]), three different series of measurements are performed. Circles indicate that a measurement takes place with result of magnitude Φ_{min} and the sign written within the circle. Doubled circles indicate the times at which $\Delta\Phi_{eff}$ is calculated.

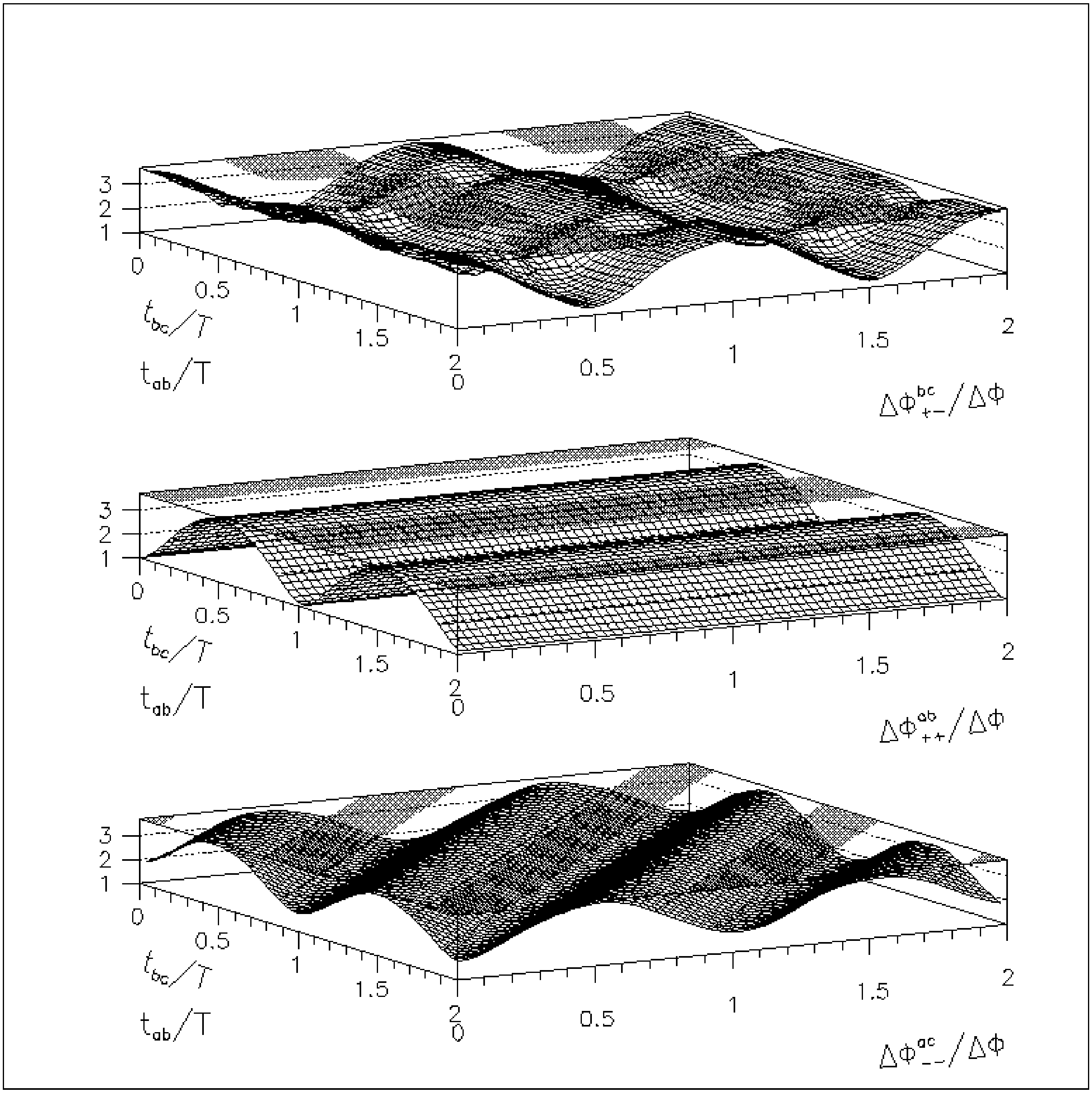
FIG. 2. Violation parameter ΔP for the temporal (*top*) and spatial (*bottom*) Bell inequality. The latter is the already-known analytical result: $\Delta P(\theta,\phi) = \sin^2(\frac{\theta}{2}) - \cos^2(\frac{\phi}{2}) - \cos^2(\frac{\theta+\phi}{2})$, whereas the former is our numerical result. A detailed analysis shows that, besides the smaller entity of the violations, in the first case the regions of violations have an asymmetrical shape in the (t_{ab}, t_{bc}) plane, as a consequence of the dependence among subsequent measurements (see Fig. 4 for details). It has been chosen the instrumental error $\Delta\Phi = 2 < \Phi_{min}$.

FIG. 3. Effective magnetic flux uncertainties $\Delta\Phi^{bc}_{+-}$, $\Delta\Phi^{ab}_{++}$, $\Delta\Phi^{ac}_{--}$, versus the measurement times t_{ab} and t_{bc} for each of the three sequences of measurements schematized in Fig. 1. On top of each graph are superimposed contour plots of the optimal regions in which the two half-wells are distinguishable, *i.e.* the effective uncertainty is less than the intra-well distance ΔL. These form periodic parallel bands with different directions in each case.

FIG. 4. Comparison between the regions of violation of the inequality (5) [shaded areas] and those in which, for all the three sequences of Fig. 1, the two half-wells remain distinguishable [small quasi-triangular zones]. The curves are evaluated for three different values of the instrumental uncertainty ($\Delta\Phi = 1, 2, 4$ as indicated). Heisenberg islands disappear for $\Delta\Phi \geq 4$; in all the other cases they have no intersection with the Bell islands.



t_{ab}
t_{bc}
I
II
III
0
T
$2T$
$(N-1)T$
t
$t_a \equiv NT$
t_b
t_c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0.5
1
1.5
2
t_{bc}/T
t_{ab}/T
0
0.5
1
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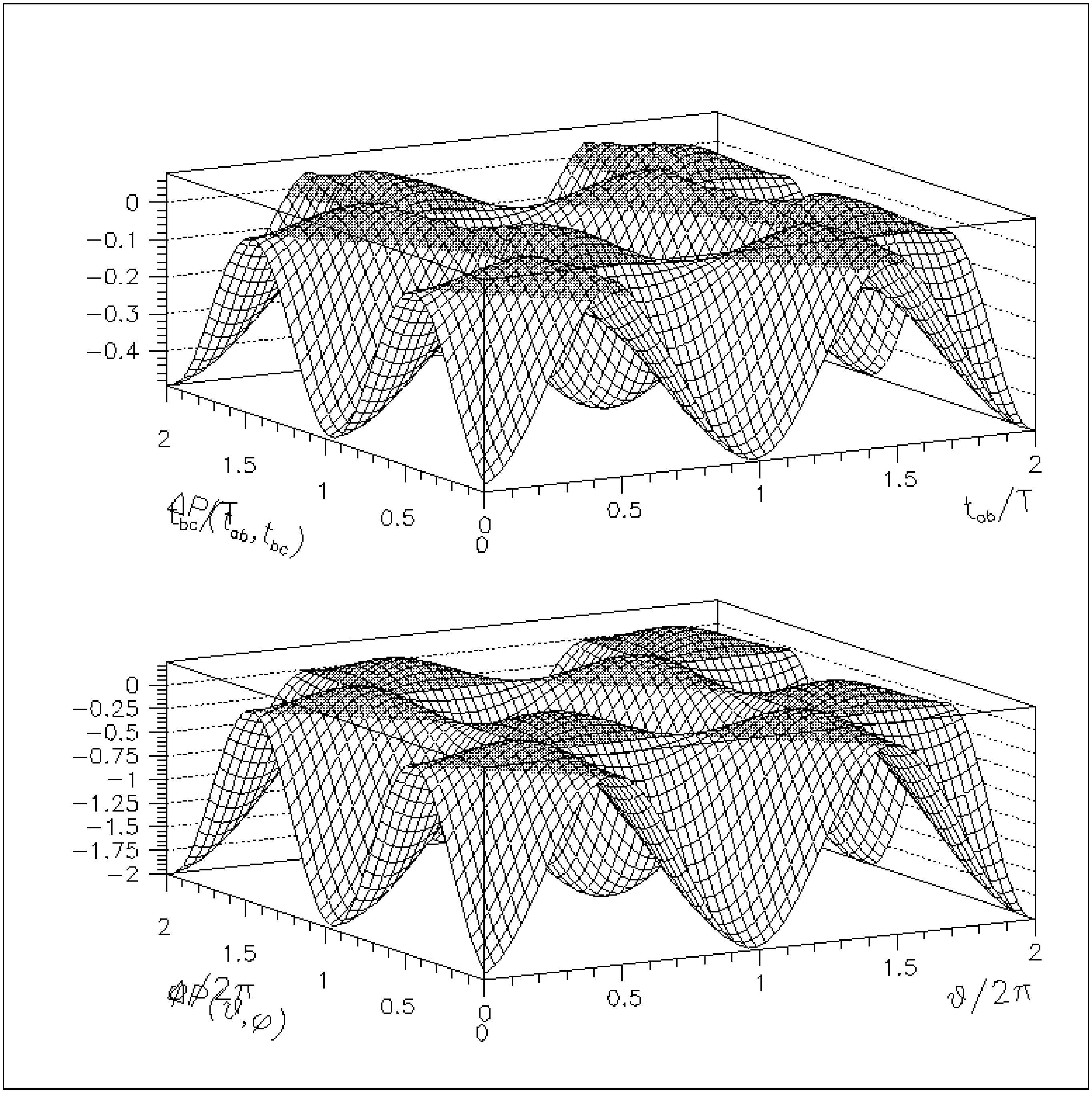
2
$\Delta\Phi^{bc}_{+-}/\Delta\Phi$
$\Delta\Phi^{ab}_{++}/\Delta\Phi$
$\Delta\Phi^{ac}_{--}/\Delta\Phi$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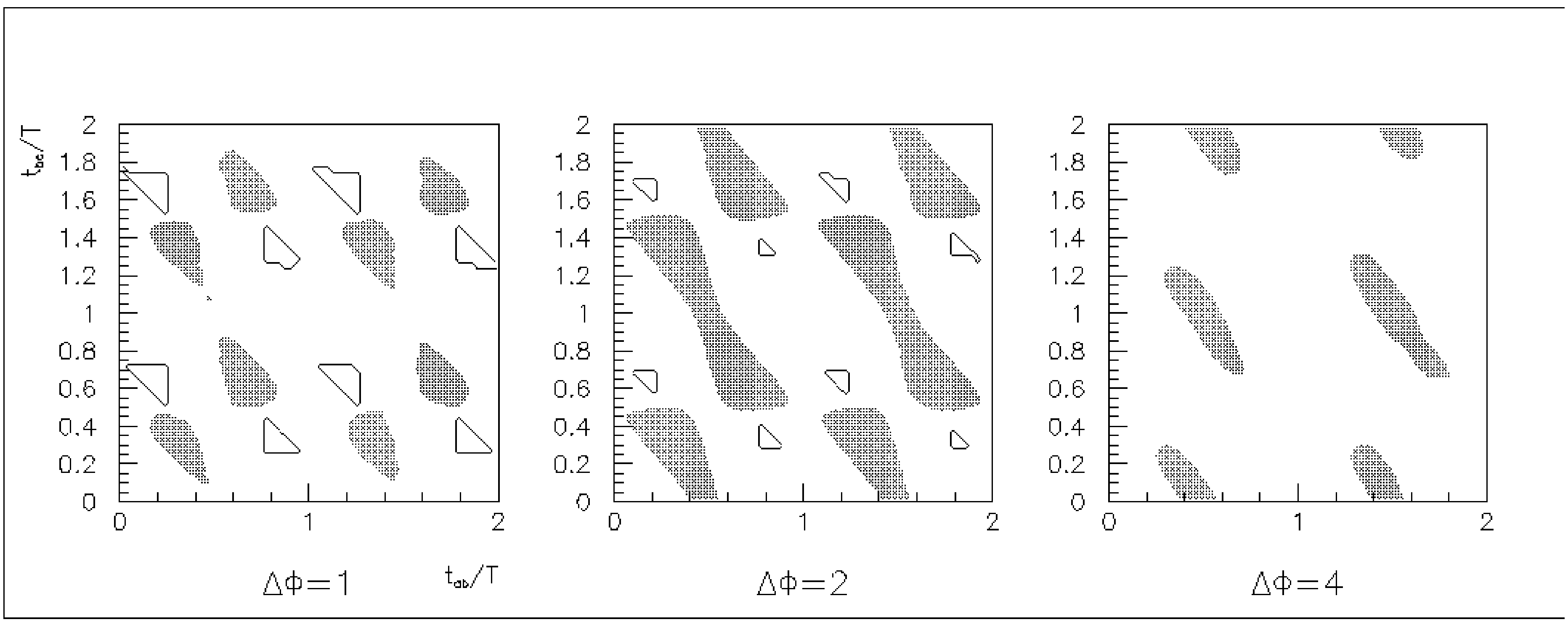
−0.4
2
1.5
1
0.5
0
0.5
1
1.5
2
t_bc/T
ΔP(t_ab, t_bc)
t_ab/T
0
−0.25
−0.5
−0.75
−1
−1.25
−1.5
−1.75
−2
φ/2π
ΔP(ϑ, φ)
ϑ/2π



t_{bc}/T
t_{ab}/T
$\Delta\Phi=1$
$\Delta\Phi=2$
$\Delta\Phi=4$